Consistent Shadow Values for Painters

Paul Centore

Abstract

A simple visual aid is presented, that can be produced in black-and-white on a piece of paper. A painter can use this aid to insure the consistency of shadow values in a painting. Consistency is necessary when an object with multiple colours, such as a shirt with red and white stripes, is seen in both light and shadow. All the values (the values of white in light, red in light, white in shadow, and red in shadow) must be correctly related to each other to make the painting convincing. Consistent values can be found by drawing a few lines on the visual aid. The scientific derivation of the visual aid is described. Finally, an application is suggested, that could heighten Adelson's "checkerblock" illusions.

Keywords — shadows, Munsell values, painting, colour, checkerblock

1 Introduction

To portray two objects in the same illumination, or one multi-coloured object, a realistic painter would like his lights and darks to be consistent with the common illumination. For example, suppose an artist is painting the red and white striped smokestack pictured in Figure 1. The left side of the smokestack is strongly lit, while its right side is in shadow, receiving less light. The artist convincingly paints the left side of the white stripes with a high-value neutral colour, such as Munsell N8. He convincingly paints the right side of the white stripes with a darker neutral, N5, and the left side of the red stripes with 5R 6/8. Figure 2A shows the progress so far.

The question now is how dark to paint the red stripes, when seen in shadow. This decision introduces the issue of consistency. A red shadow colour, that is believable when viewed in relation to the red colour in light, might not be believable when viewed alongside the lit and shadowed whites, which are in the same lighting

Figure 1: A Real-World Example of Consistent Shadow Values

condition. Figures 2B and 2C show two possible shadow colours for the red. The shadow colour in 2B is lighter than the shadow colour in 2C. The colours are shown, away from the white and its shadow colour, at the bottom of 2B and 2C.

Seen without the white stripes for a comparison, the shadow colours in 2B and 2C are both plausible. The shadow in 2C would occur with a very directed light source, while 2B would occur under more diffuse lighting conditions. Seen in the context of the smokestack, however, in the top of 2B and 2C, either shadow colour is inconsistent. The error is most obvious in 2C, where the shadowed white stripes seem to glow, relative to the shadowed red stripes. This effect occurs because the eye cannot determine a set of lighting conditions that is consistent for both the red and the white stripes.

The important variable in this example is Munsell value: if the shadow's value is decided, and the colour in light is known, then References [1] and [2] show how to find a shadow's hue and chroma. The hue and chroma of the two shadow colours in 2B and 2C were chosen so that the colours are both in the shadow series of 5R 6/8, and have values 5 and 3. The key painting decision, then, is Munsell value.

This article presents a simple visual aid, Figure 3, that an artist can use to determine consistent shadow values, so that different objects, or different parts of the same object, can be portrayed in the same lighting. This aid would be particularly useful when painting from imagination, or when combining studies, made in different

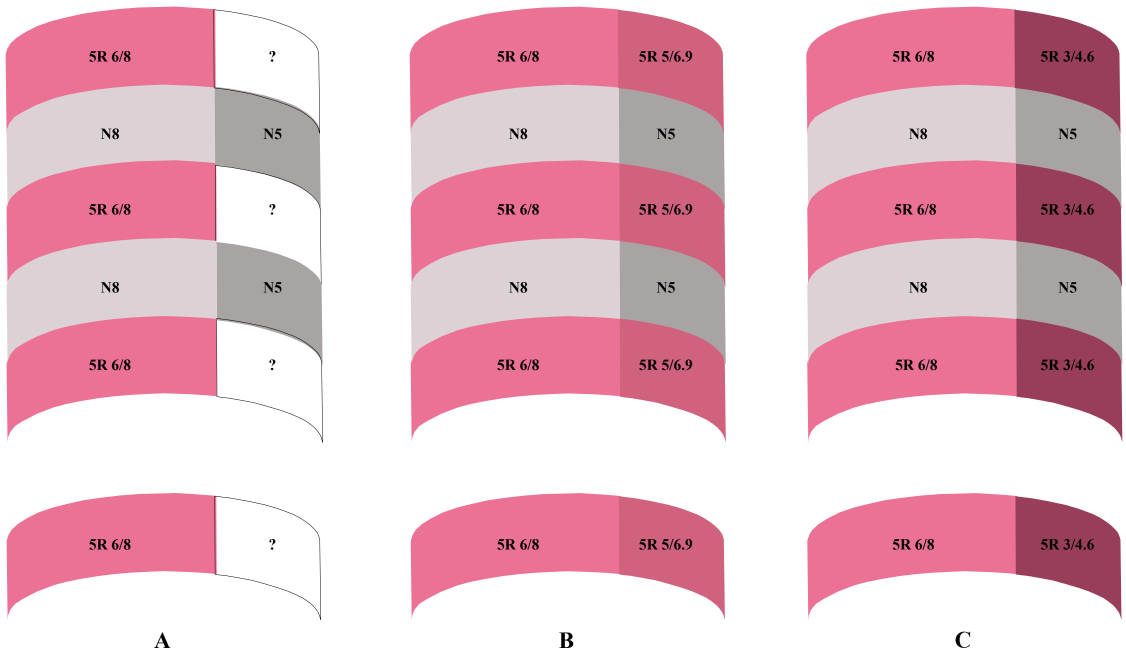

Figure 2: Possible Approaches to Shadow Values

lighting situations, into one painting. The physical and mathematical derivation of the visual aid is described. Also, a potential application is suggested, that could heighten Edward Adelson's "checkerblock" illusions.

2 The Munsell System

Albert Munsell devised his colour specification system for painters and visual artists. It classifies surface colours by three perceptual attributes that are basic to painting: hue, value and chroma.

Hue is universally understood. It says whether a colour is red, yellow, purple, etc. Munsell designates 10 basic hues: R (red), YR (yellow-red, or orange), Y (yellow), GY (green-yellow), G (green), BG (blue-green), B (blue), PB (purple-blue), P (purple), and RP (red-purple). Each basic hue is further subdivided into 4 steps, denoted with a prefix. For example, the four greens are denoted 2.5G, 5G, 7.5G, and 10G. 2.5G is a yellower green, that is closer to GY than it is to BG. 10G is a bluer green, that is closer to BG than it is to GY. In all, then, the Munsell system specifies 40 hues (4 steps for each of the 10 basic hues). These 40 hues are equally spaced perceptually. One could interpolate any desired amount between two adjacent hues.

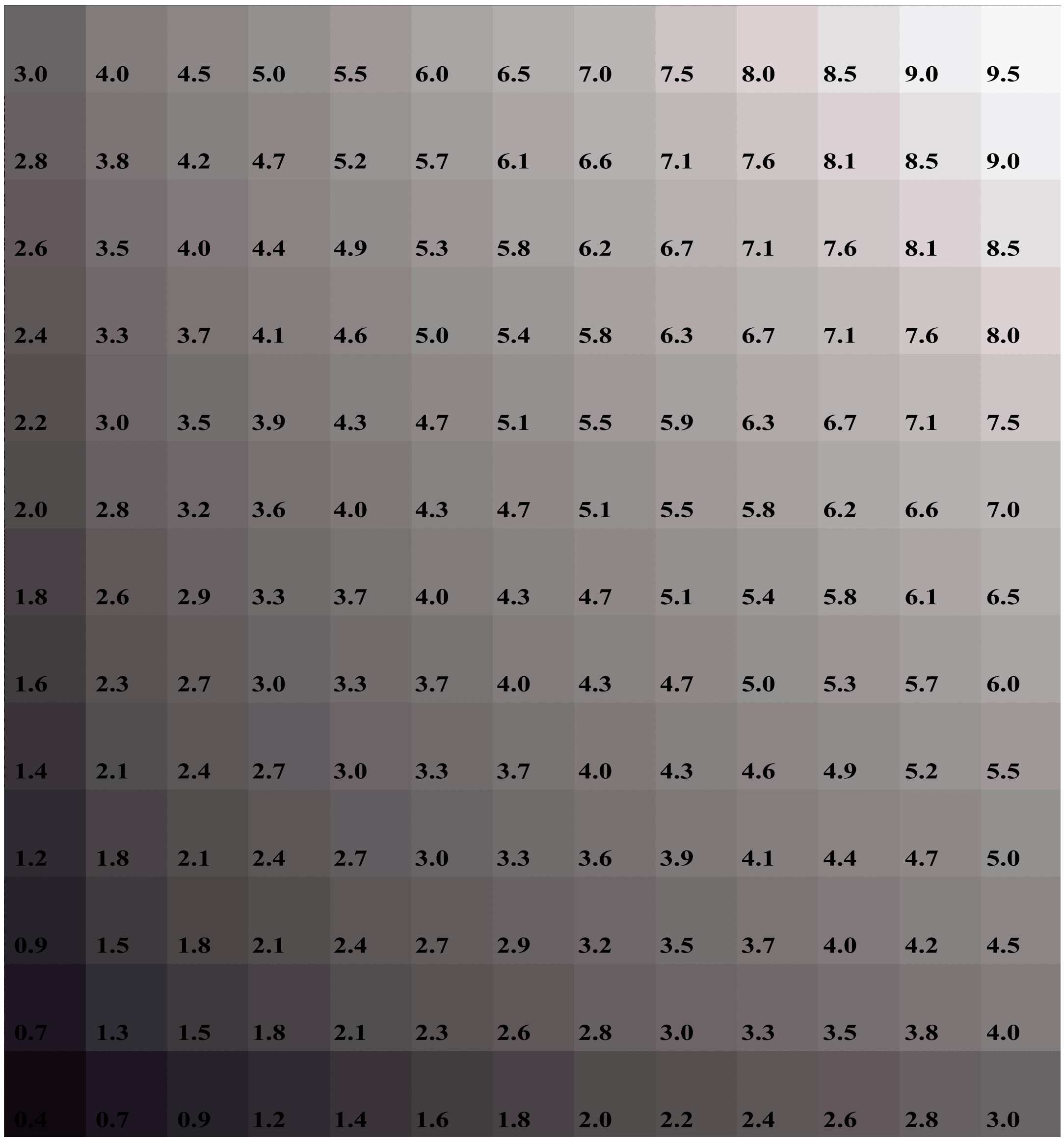

Figure 3: Visual Aid for Consistent Shadow Values

For example, the hue 6GY is a yellowish green that is between 5GY and 7.5GY, but closer to 5GY. White, black, and greys are not considered hues in the Munsell system. Rather, they are designated N, for “neutral.”

Many different colours can have the same hue. Figure 4, for example, shows the “hue leaf” for 6GY, a set of colours all of which have hue 6GY. The different colours

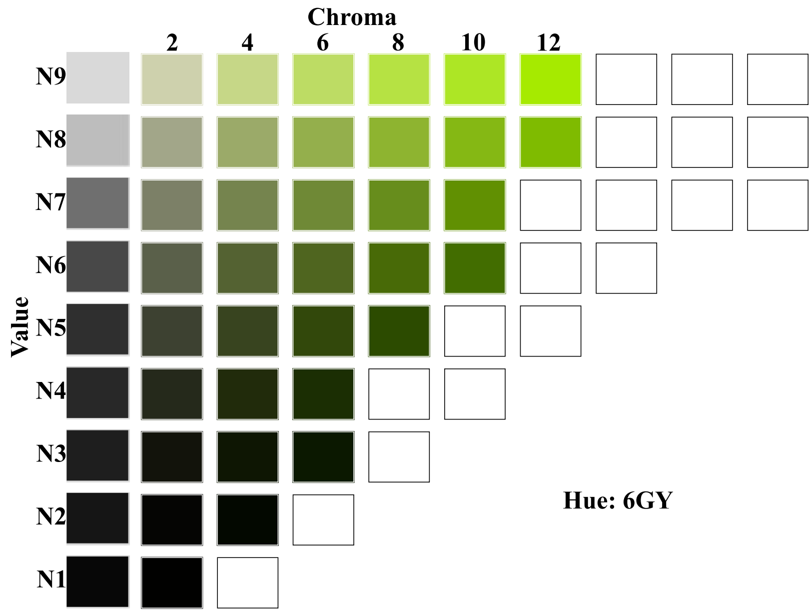

Figure 4: The Hue Leaf for 6GY in the Munsell System

within a hue leaf are specified further by value and chroma. The empty boxes indicate colours that are in the Munsell system, but that are beyond the gamut of the printing process used to produce the figure. The hue leaf shades smoothly into the neutral axis, consisting of greys, shown on the left.

Munsell value designates how light or dark a colour is. The theoretically darkest black has a value of 0, and is denoted N0. The theoretically lightest white has a value of 10, and is denoted N10. Between N0 and N10 are 9 progressively lighter greys, denoted N1 through N9. The spacing between the greys is perceptually equal. All colours have a Munsell value, not just the neutrals. For example, there are light blues and dark blues. A blue with value 8.5 has the same lightness as N8.5.

Munsell chroma refers to how intense, or saturated, a colour is. For example, a lemon is an intense yellow, while masking tape is a dull yellow. A dull colour is closer to a neutral grey than an intense colour. The Munsell system denotes chroma numerically. Greys have chroma 0. A colour with a chroma of 10 or higher is generally perceived as saturated. Colours of low chroma, say 4 or less, are perceived as subdued, with a high grey content.

The Munsell notation for a colour takes the form H V/C, where H stands for hue, V stands for value, and C stands for chroma. For example, the colour 10R 9/6 would be a very light (V is 9), moderately intense (C is 6), orangish red (H is 10R).

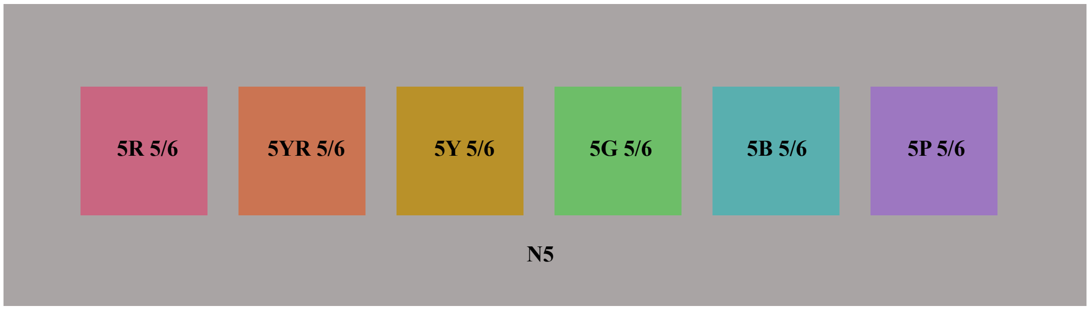

Figure 5: "Vibration" When Colours Have the Same Value

A colour with chroma 0 is a neutral grey, which is denoted NV, where V stands for value. For example, N5 is a grey that is midway between white and black.

To obtain consistent shadow values, a painter must visually extract a colour's Munsell value. Any two colours that are the same value, but of significantly different hue or chroma, will seem to "vibrate" when placed next to each other. Figure 5 shows some examples, in which a moderately chromatic colour is placed on a grey of the same value. The vibration occurs because value is a more important perceptual cue than hue or chroma. Hue and chroma differences are weaker perceptual cues, and they are overlooked at first if there is no value difference. The human eye does a double-take, as it belatedly realizes that the center colour is in fact different from its surround of the same value. To evaluate a colour's value, then, it should be viewed against greys of various known values. It will vibrate visually when seen against a grey of the same value.

3 A Visual Aid for Consistent Shadow Values

Figure 3 shows a visual aid that helps a painter obtain consistent shadow values. The aid consists of rows of different sequences of greys. Each row shows how the first row would look in some degree of shadow, with darker shadows occurring nearer the bottom.

To use this aid, first determine the Munsell values of the local colours of interest, and mark those values on the top row of Figure 3. Then draw vertical lines down the figure, from the marked values. Figure 6 shows an example, in which the red and white stripes in a smokestack have local values 6 and 8, respectively, as illustrated in Figure 2. In a scene, different amounts of light would impinge on different regions of the smokestack. Suppose the lighting on the shadowed region caused the white stripes to appear to have value 5. Then the painter should identify the cell, or point within a cell, on the right-hand vertical line (which corresponds to the white stripes) that corresponds to value 5, and draw a horizontal line through it, as shown in the figure. The horizontal line in this example crosses the left-hand vertical line (which corresponds to the red stripes) at a value of 3.7, so consistency can be achieved by painting the red stripes with that value.

Once the shadow's value is known, Reference [1] can be used to determine its chroma, which in this case is 5.4. Figure 7 repeats Figure 2, with the uncertain shadow colours filled in as 5R 3.7/5.4. The shadows in 2A have consistent values: the red and white stripes appear to be in the same degree of shadow (on the right side), and at the same level of illumination (on the left side). Figure 7A should be compared with Figures 7B and 7C, where it is difficult to interpret the stripes in shadow and in illumination.

4 Derivation of the Shadow Value Aid

The aid was developed mathematically, from calculations involving Munsell value V, and luminance factor Y. The conversion between them is given by a quintic polynomial that appeared originally in the Munsell renotation [3], and in a rescaled version in an ASTM standard [4].

Every point of a scene has a luminance factor, which measures the amount of light from that point (weighted with respect to human sensitivity to different wavelengths), which impinges on a painter's eye. Every point of a painting of that scene has a Munsell value. The Munsell value, and indeed the entire Munsell specification, depends only on the reflectance characteristics of the paint at a particular point. In a realistic painting, each point on the canvas corresponds to a particular point in the scene. The relationship between the luminance factor and Munsell value at corresponding points is not completely understood. A painter can "key" the values, within some limits, and still produce a convincing painting, much as a photographer has some latitude with exposure time. Nevertheless, there must be some consistency: darker parts of the scene should be painted darker than lighter parts of the scene, and by the same relative amount.

The luminance factor of a part of an object in a scene depends on two other factors: the reflectance properties of that part, and the illumination impinging on that part. A shadow occurs when there is less illumination on some parts of the object than on others. In the smokestack, for example, there is less illumination on the right side than on the left side. Formally, the light side of the smokestack might be receiving light whose spectral power distribution (SPD) is given by $E(\lambda)$, while

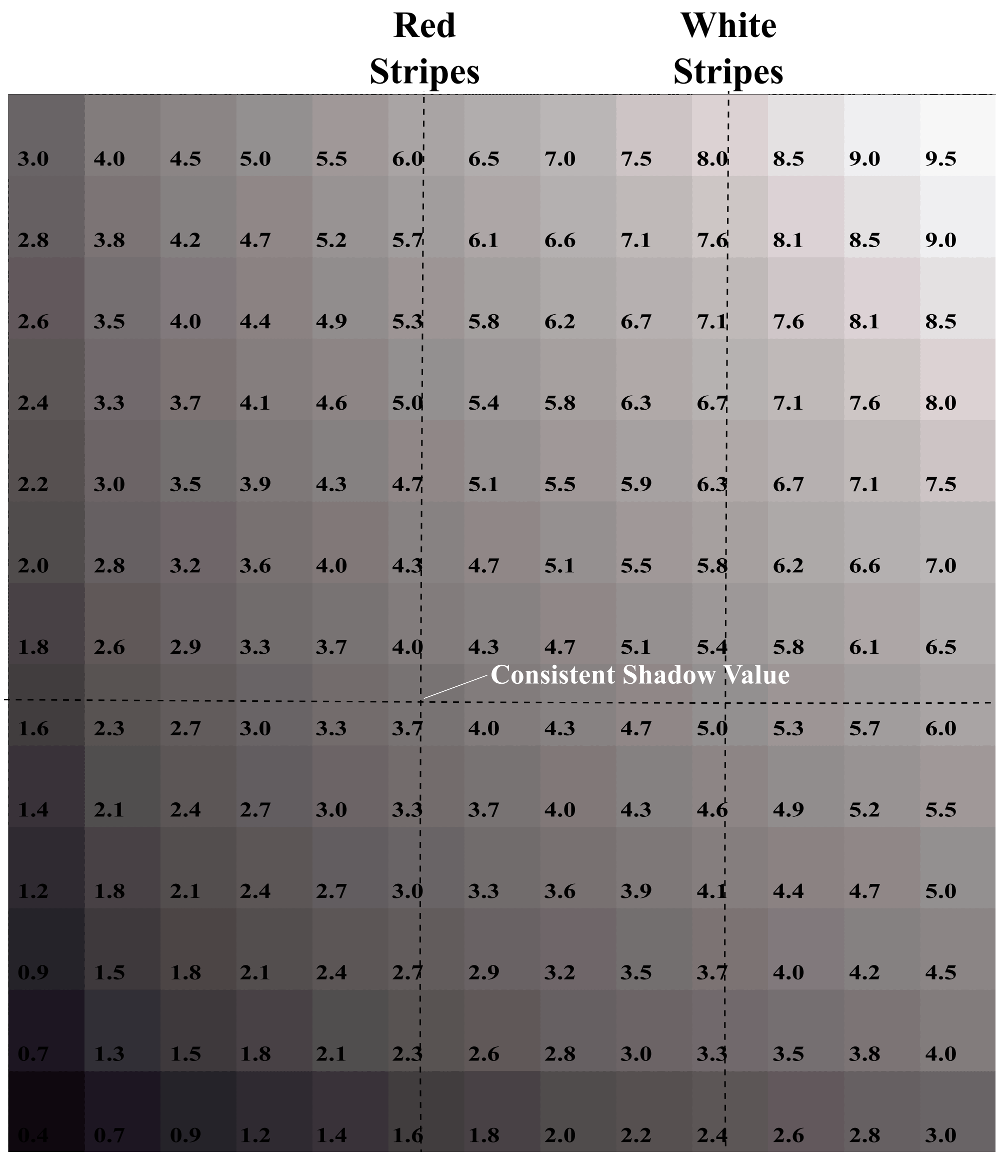

Figure 6: Example Use of Visual Aid, for Painting Red and White Stripes

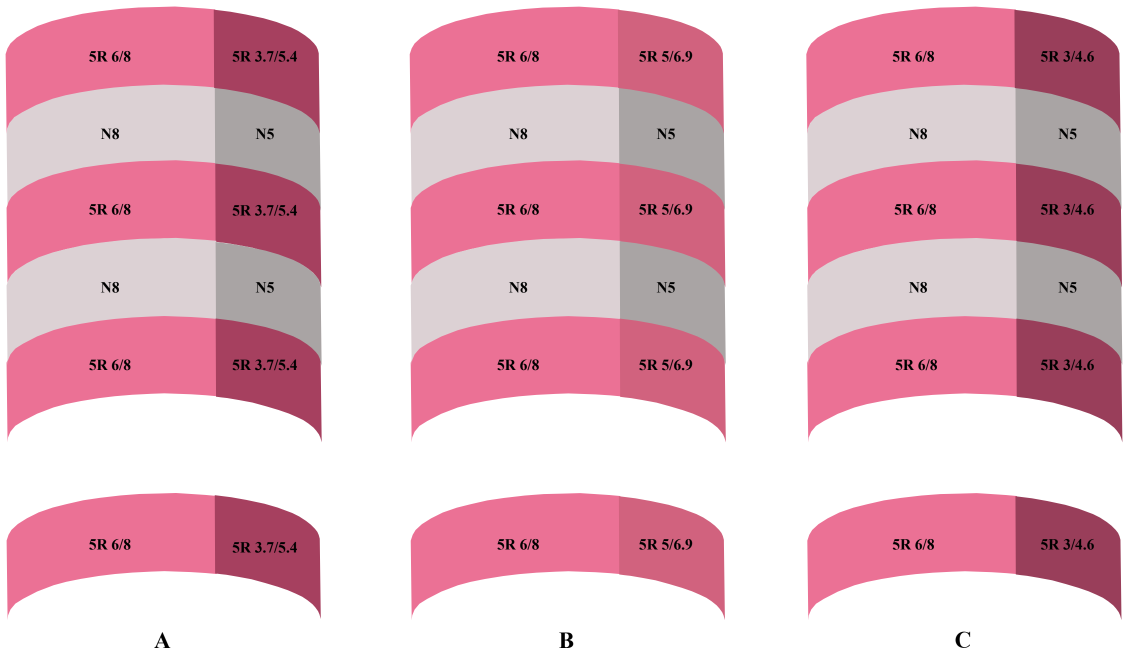

Figure 7: Consistent Shadow Value Example

the shadowed side is receiving light whose SPD is $kE(\lambda)$, where k is positive but less than 1. The reflectance properties of the part in question are described by a reflectance function, $\rho(\lambda)$, where λ is a wavelength in the visible spectrum, which stretches approximately from 400 nm to 700 nm. ρ can take on values in the interval from 0 to 100%.

The luminance factor Y_L for the part in light is given by

$$Y_L = \frac{\int_{400}^{700} \bar{y}(\lambda)\rho(\lambda)E(\lambda)d\lambda}{\int_{400}^{700} \bar{y}(\lambda)E(\lambda)d\lambda}, \quad (1)$$

where $\bar{y}$ is the photopic luminous efficiency function [5]. The luminance factor Y_S for the part in shadow is given by

$$Y_S = \frac{\int_{400}^{700} \bar{y}(\lambda)\rho(\lambda)kE(\lambda)d\lambda}{\int_{400}^{700} \bar{y}(\lambda)E(\lambda)d\lambda}. \quad (2)$$

From Equations 1 and 2, it follows that

$$Y_S = kY_L. \quad (3)$$

A notable fact is that Equation 3 holds independently of the reflectance function $\rho(\lambda)$. In the smokestack example, the red and white stripes have very different reflectance functions. Despite this difference, Equation 3 applies to both of them: the ratio of the luminance factor in shadow, to the luminance factor in light, is the same k, for both the red and the white stripes. Physically, then, shadow values can be achieved by multiplying the luminance factor in light by the same constant, for all colours in the same light and shadow configuration.

Perceptually, however, multiplication by the same constant is not correct. The reason is that the perception of lightness, which is codified in the Munsell value, is not a linear function of the luminance factor. Instead, it is given by the quintic equation [4]:

$$Y = 0.00081939V^5 - 0.020484V^4 + 0.23352V^3 - 0.22533V^2 + 1.1914V, \tag{4}$$

where V is the Munsell value and Y is expressed as a percentage. Munsell values must be converted to luminance factors, after which multiplicative constants can be determined and applied. Then the resulting luminance factors can be converted back to Munsell values.

In the example in Figure 2, the white stripes in light and shadow have Munsell values 8 and 5, respectively. These correspond to luminance factors of 57.6% and 19.3%. The ratio, k, of these luminance factors is 19.3/56.7, which is about 0.34. The red stripe in light has a Munsell value of 6, which corresponds to a luminance factor of 29.3%. The luminance factor of the red stripe in shadow should therefore be 0.34 times 29.3%, which is about 10.0%. Inverting Equation 4 converts a luminance factor of 10.0% to a Munsell value of 3.7. The red shadow, then, should be painted with Munsell value 3.7. Of course, the same result was obtained from using the visual aid in Figure 3.

The calculations behind the visual aid should now be clear. The first row of Figure 3 is a selection of Munsell values for colours in light. The second row is the Munsell values of the first row, when viewed in shadow defined by a constant k. The second row was scaled by requiring the cell on the far right to have Munsell value 9.0. Since the cell on the far right in the first row has Munsell value 9.5, the method of the previous paragraph gives a k of 0.87. The luminance factor of each cell in the second row is therefore 0.87 times the luminance factor of the cell above it in the first row. The luminance factors have been converted back to Munsell values, which are shown on the visual aid. Further rows were calculated by the same method, after choosing a set of values for the far right cells, that should provide a fine enough resolution for painting. Interpolation might be required when painters use the visual aid. Since a difference of less than 0.25 Munsell value steps is usually indistinguishable, however,

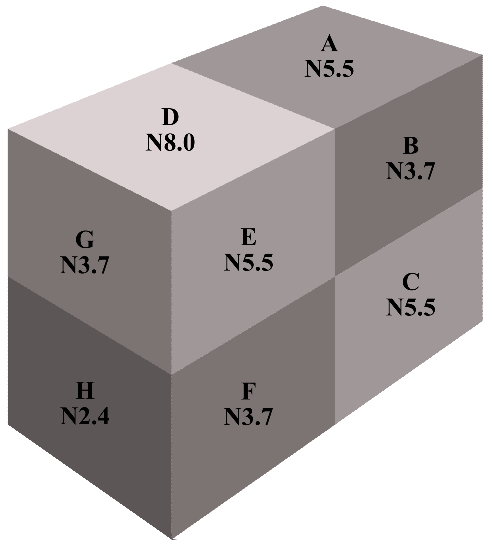

Figure 8: Checkerblock Ilusion

and differences of less than 0.5 Munsell value steps are difficult for painters to mix, the aid should be sufficiently accurate.

5 Applications to Adelson's Checkerblocks

In 1993, Edward Adelson introduced his well-known "checkerblock" illusions [6] , an example of which is shown in Figure 8. The illusion is that diamond A appears darker than diamonds C and E, when in fact they all have the same lightness. A plausible explanation is that the viewer interprets the figure as a piece of a 2-by-2 checkerboard, that has been extended into the third dimension. The diamonds are therefore not seen as flat shapes on a two-dimensional viewing surface, but as parts of a solid that exhibits shadowing. Figure 11.1 (c) of Reference [7] shows a possible arrangement of shadows. The viewer discounts the shadowing, in order to arrive at local colours. The simplest conclusion is that diamonds D, E, and G are facets of one cube, which has a uniform light colour, while A and B are facets of another cube, which has a uniform dark colour. Diamond E is therefore perceived as lighter than diamond A, even though their lightnesses are equal.

The checkerblock illusion would probably be most effective if the lightnesses of the diamonds were chosen to give consistent shadow values. For example, diamonds

B and E are shadowed versions of A and D, respectively. In most realistic situations, the amount of light hitting one face will not vary appreciably, so B and E should have the same *degree* of shadow, relative to A and D. If the factor k_1 corresponds to this degree of shadow, then

$$\begin{aligned} Y_B &= k_1 Y_A, &(5)\\ Y_E &= k_1 Y_D. &(6) \end{aligned}$$

Similarly, diamonds G and H are shadowed versions of E and F, respectively, so

$$\begin{aligned} Y_G &= k_2 Y_E, &(7)\\ Y_H &= k_2 Y_F. &(8) \end{aligned}$$

The original illusion had $Y_A = Y_C$. Since C and E are presumably the same colour, we therefore have

$$Y_A = Y_E = Y_C. \qquad (9)$$

To make the illusion simpler and even more compelling, we can also require G and F to have the same lightness, getting

$$Y_G = Y_F = Y_B. \qquad (10)$$

Equations 5 through 10 can be combined to give a set of relationships that depend only on Y_A and Y_D:

$$\begin{aligned} Y_C &= Y_E = Y_A, &(11)\\ Y_B &= Y_F = Y_G = \frac{Y_A^2}{Y_D}, &(12)\\ Y_H &= \frac{Y_A^3}{Y_D^2}. &(13) \end{aligned}$$

Given initial choices for Y_A and Y_D, Equations 11 through 13 can be used to determine all the lightnesses in a checkerblock. The lightnesses can be expressed as either luminance factors or Munsell values. Figure 8 shows the checkerblock that results when Munsell values of 5.5 and 8.0 are chosen for diamonds A and D, respectively. The colours are all chosen to be neutral greys.

It is unclear from Reference [6] what values were used for experiments with the original checkerblock. The article mentions that the illusions were displayed on computer screens. The luminances for some other illusions are given in milliLamberts.

No mention, however, is made of the ambient viewing conditions, so luminance factors cannot be calculated. As Stevens and Stevens point out [8], perceived luminance differences vary when the eye is adapted to different viewing conditions. The quintic relationship given by Equation 4 is valid only when the lighting level is approximately that of indirect daylight. The perceived shadow relationships in the same illusion will therefore be different when viewed in a dim room and in a well-lit room. The instructions given here will produce illusions that are effective when printed or painted, and viewed under everyday working conditions. The consistent shadow values should heighten the checkerblock illusion by making the three-dimensionality more convincing.

References

1. Paul Centore, "Shadow Colours for Painters," 2012
2. Paul Centore, "Shadow Series in the Munsell System," Color Research & Application, Vol. 38, No. 1, pp. 58-64, February 2013.
3. Sidney Newhall, Dorothy Nickerson, & Deane B. Judd. "Final Report of the O. S. A. Subcommittee on the Spacing of the Munsell Colors," JOSA, Vol. 33, Issue 7, pp. 385-418, 1943.
4. ASTM Standard D 1535-08. "Standard Practice for Specifying Color by the Munsell System," 2008.
5. Deane B. Judd, "The 1931 I. C. I. Standard Observer and Coordinate System for Colorimetry," JOSA, Vol. 23, pp. 359-374, October 1933.
6. Edward H. Adelson, "Perceptual Organization and the Judgment of Brightness," Science, Vol. 262, pp. 2042-2044, 24 December 1993.
7. E. H. Adelson & A. P. Pentland, "The perception of shading and reflectance," Chapter 11 in *Perception as Bayesian Inference*, eds. D. Knill & W. Richards, Cambridge University Press, 1996.
8. J. C. Stevens & S. S. Stevens, "Brightness Function: Effects of Adaptation," JOSA, Vol. 53, Issue 3, pp. 375-385, 1963.

